Problem 13.10

What is the acceleration due to gravity of an object that is three earth radii above the earth's surface?

Although this looks like a simple N.S.L. problem, it has a very clever twist to it. When you are sitting on the earth's surface, the magnitude of the gravitational force on you is:

$$\frac{Gm_e m_{you}}{R_e^{\,2}}$$

where R_e is the radius of the earth (that is the distance between the *center of mass* of the two bodies) and m_e is the mass of the earth. Both of those quantities are known, so we can write:

$$\left(\frac{Gm_e}{R_e^{\,2}}\right)m_{you} = \left(\frac{\left(6.67\text{x}10^{-11}\ \text{kg}\right)\left(5.98\text{x}10^{24}\ \text{kg}\right)}{\left(6.37\text{x}10^{6}\ \text{m}\right)^2}\right)m_{you}$$
$$= m_{you}\left(9.83\ \text{m/s}^2\right)$$

This is where the idea that your weight is due to the earth's gravitational effect on you, and is equal to "mg." The constant "g" is just the calculated 9.8 m/s/s.

The point is, we can either dutifully write out

$$\sum F:$$
$$\frac{Gm_e m}{\left(4R_e\right)^2} = ma$$
$$\Rightarrow\ a = \frac{Gm_e}{\left(4R_e\right)^2}$$

Then put in the numbers, or we can be clever and write:

$$a = \frac{Gm_e}{\left(4R_e\right)^2} = \frac{1}{16}\left(\frac{Gm_e}{\left(R_e\right)^2}\right)$$
$$= \frac{1}{16}\left(9.83\ \text{m/s}^2\right) = .614\ \text{m/s}^2$$